Weighted Digraphs And Graphs

A weighted digraph is a pair \((V, w)\) where \(V\) is a finite set of vertices and \(w\) is a function that assigns to each pair \((x, y)\) of vertices either a positive integer or \((\infty)\). The function \(w\) is called the weight function, and its value \(w(x, y)\) can be interpreted as the cost (or time or distance) for moving directly from \(x\) to \(y\). The value \(w(x, y) = \infty\) indicates that there is no edge from \(x\) to \(y\).

A weighted graph is a weighted digraph \((V, w)\) whose weight function \(w\) is symmetric, that is, \(w(y, x) = w(x, y)\) for all \(x, y \in V\). Just as every digraph has an embedded graph, every weighted digraph has an embedded weighted graph \((V, w')\) and an embedded (unweighted) digraph. The weight function for the embedded weighted graph can be defined as \(w'(x, y) = \min\{w(x, y), w(y, x)\}\), where \(w\) is the weight function of the weighted digraph. The vertex set for the embedded digraph can be defined as \(E = \{(x, y) : w(x, y) < \infty\}\).

The properties described above for digraphs and graphs apply to weighted digraphs and weighted graphs. In addition there are some extended properties that depend upon the underlying weight function in the obvious manner. For example, the weighted path length is the sum of the weights of the edges along the path. And the shortest distance from \(x\) to \(y\) would be the minimum weighted path length among all the paths from \(x\) to \(y\).

A Weighted Digraph and Its Embedded Structures

Figure 15.20 shows a weighted digraph together with its embedded weighted graph, its embedded digraph, and its embedded graph. The weights are shown on the edges.

In graph \(G_1\) the weighted path length of the path \(cabd\) is \(|cabd| = 2 + 3 + 2 = 7\), and the shortest distance from \(c\) to \(d\) is 6 (along the path \(cad\)). But in graph \(G_2\) that shortest distance is 1 (along the path \(cd\)).

Figure 15.21 shows the adjacency matrix, the incidence matrix, and the adjacency list for graph \(G_1\).

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Source: Schaum's Outline of Data Structures with Java 2nd Edition