Euler Paths And Hamiltonian Cycles

An euler path in a graph is a walk that includes each edge exactly once. An euler cycle is a closed walk that includes each edge exactly once. An eulerian graph is a graph that has an euler cycle.

Note that euler paths and cycles need not have distinct vertices, so they are not strict paths.

Euler Paths and Cycles

In the graph in Figure 15.22, the closed walk acedabefbcfa is an euler cycle. So this is an eulerian graph. Note that every vertex in this graph has degree 4, and its 12 edges are partitioned into three circles. As the Theorem reports, each of these two properties will always guarantee that the graph is eulerian.

![Graph diagram](image)

**Figure 15.22** An eulerian graph

**Theorem.** Eulerian Graphs

If G is a connected graph, then the following conditions are equivalent:

1. G is eulerian.
2. The degree of each vertex is even.
3. The set of all edges of G can be partitioned into cycles.

A hamiltonian path in a graph is a path that includes each vertex exactly once. A hamiltonian cycle is a cycle that includes each vertex exactly once. A hamiltonian graph is a graph that has a hamiltonian cycle.

Unfortunately, there is no simple characterization like Theorem for hamiltonian graphs. In fact, the problem of finding such a simple characterization is one of the big unsolved problems in computer science.

Hamiltonian Graphs

In Figure 15.23, the graph on the left is hamiltonian. The graph on the right is not; it has a hamiltonian path, but no hamiltonian cycle.
Figure 15.23  A graph with a hamiltonian cycle, and one with without one

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Source: Schaum's Outline of Data Structures with Java 2nd Edition
by John R. Hubbard